String-Inspired Gravity Coupled to Yang-Mills Fields*

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ABSTRACT

String-inspired 1+1-dimensional gravity is coupled to Yang-Mills fields in the Cangemi-Jackiw gauge-theoretical formulation, based on the extended Poincaré group. A family of couplings, which involves metrics obtainable from the physical metric with a conformal rescaling, is considered, and the resulting family of models is investigated both at the classical and the quantum level. In particular, also using a series of Kirillov-Kostant phases, the wave functionals that solve the constraints are identified.

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I Introduction

The problem of constructing a consistent quantum theory of gravity has proven to be extremely hard. It appears that, once there is a quantum dynamics for the geometry, very few of the tools used in the quantization of theories in a background geometry are available.

One appealing possibility to tackle the problem is the one of casting gravity in a gauge theoretical formulation, so that we would be able to draw from the experience gained in the many successful quantizations of gauge theories. Some of the most interesting proposals providing such formulations are the Ashtekar formulation of Einstein gravity [1], the Poincaré gravities [2, 3], the Chern-Simons gravity [4], and, most recently, the Cangemi-Jackiw [5] gauge theoretical reformulation, based on the extended Poincaré group, of string-inspired (1+1-dimensional) gravity [6].

The Cangemi-Jackiw approach to 1+1-dimensional quantum gravity has been used in investigations of pure gravity [5], gravity coupled to point particles [7, 8], and gravity coupled to scalar matter fields[9]; however, the analysis of gravity coupled to gauge fields, which is the objective of the present paper, had not been previously performed. Obviously, for gauge theoretical formulations of quantum gravity the coupling to gauge fields can be very interesting; most importantly, one expects simplifications (with respect to corresponding non gauge theoretical formulations[10, 11, 12]) to arise, allowing to make substantial progress.

One important aspect of our analysis is that we consider different ways to couple gauge fields to gravity. We consider a family of couplings involving metrics that can be obtained from the Cangemi-Jackiw gauge metric with a conformal rescaling, so, in particular, we have as limiting cases the minimal coupling via the gauge metric itself and the minimal coupling via the physical metric, as done in Ref. [8]. (The definitions of the gauge metric and the physical metric are reviewed in the following section.) In particular, the investigation of the coupling via the physical metric might be relevant to the understanding of the nature of the divergencies encountered in Ref. [8] in relation to the Poincaré coordinates. A crucial point is that in the case of gauge-theoretical gravity coupled to N point particles [8] one is naturally lead to the introduction of N sets of Poincaré coordinates associated to the actual coordinates of the particles (upon appropriate gauge choice the Poincaré coordinates are indeed the coordinates of the particles), whereas the coupling to fields always involves one set of Poincaré coordinates, which however are then functions taking values on the entire 1+1-dimensional space-time.
Throughout the paper, the abelian limit of the results that we obtain for arbitrary Yang-Mills fields within the gauge-theoretical formulation are compared to the corresponding results obtained within the geometric approach of Ref.[12], in which only the coupling to abelian gauge fields was considered.

Before proceeding to the quantization of our models, which is the primary objective of this paper, for completness in the next two sections we review the Cangemi-Jackiw gauge theoretical formulation and analyze our models at the classical level.

II Gauge formulation of lineal gravity

The (geometrical) action of string-inspired gravity [6] is given by

$$I = \frac{1}{2\pi\kappa}\int d^2x \sqrt{-g^P} e^{-2\phi}(R(g^P) + 4{g^P}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \lambda). \quad (2.1)$$

where λ is the cosmological constant and ϕ the dilaton field.† The introduction of the new variables

$$g_{\mu\nu} = e^{-2\phi} g^{P}_{\mu\nu} \quad \text{and} \quad \eta = e^{-2\phi} \tag{2.2}$$

transforms the action (2.1) into a simpler expression

$$I = \frac{1}{2\pi\kappa} \int d^2x \sqrt{-g}(\eta R(g) - \lambda), \tag{2.3}$$

which can be reformulated as a gauge theory [2, 5, 13]. (In the following, in order to avoid ambiguities, we shall refer to $g^{P}_{\mu\nu}$ as the *"physical"* metric, and to $g_{\mu\nu}$ as the *"gauge"* metric.) In particular, a gauge theoretical formulation of the action (2.3) can be given by using the 4-parameter extended Poincaré group in 1+1 dimensions [5, 7, 9], whose Lie algebra reads

$$[P_a, P_b] = \epsilon_{ab} I\,, \qquad [P_a, J] = \epsilon_a{}^b P_b \tag{2.4}$$
$$\tag{2.5}$$

Here, P_a and J are the usual translation and boost generators, while I is the central element. Such extension arises naturally in two dimensions if one allows non-minimal gravitational coupling, as pointed out in Ref.[5].
The field, which will describe gravity, is now introduced as a connection one-form that takes values in the Lie algebra

$$B_\mu = e^a_\mu P_a + \omega_\mu J + a_\mu I. \tag{2.6}$$

e^a and ω are the *zweibein* and the spin connection respectively; the potential a_μ is, instead, related to the volume form [5]. The connection defined in (2.6) transforms according to the adjoint representation. In components the transformation is

$$e^a_\mu \to (\Lambda^{-1})^a{}_b(e^b_\mu + \epsilon^b{}_c\theta^c\omega_\mu + \partial_\mu\theta^b), \tag{2.7}$$
$$\omega_\mu \to \omega_\mu + \partial_\mu\alpha, \tag{2.8}$$
$$a_\mu \to a_\mu - \theta^a\epsilon_{ab}e^b_\mu - \frac{1}{2}\theta^a\theta_a\omega_\mu + \partial_\mu\beta + \frac{1}{2}\partial_\mu\theta^a\epsilon_{ab}\theta^b, \tag{2.9}$$

where we have parameterized the gauge transformation as follows

$$U = \exp(\theta^a P_a)\, \exp(\alpha J)\, \exp(\beta I) \tag{2.10}$$

and $\Lambda^a{}_b$ is the Lorentz transformation matrix

$$\Lambda^a{}_b = \delta^a{}_b \cosh\alpha + \epsilon^a{}_b \sinh\alpha. \tag{2.11}$$

The field strength $\mathcal{R}$ can be now computed from its definition

$$\begin{aligned}\mathcal{R} &= dB + [B, B]\\ &= (de^a + \epsilon^a{}_b\omega \wedge e^b)P_a + d\omega J + (da + \frac{1}{2}\epsilon_{ab}e^a \wedge e^b)I.\end{aligned} \tag{2.12}$$

†Notation: the signature of the metric tensor $g^{P}_{\mu\nu}$ is assumed to be $(1, -1)$. The Latin indices $a,\ b,\ c\ldots$ run over a tangent space where the flat Minkowski metric $h_{ab} = \text{diag}(1, -1)$ is defined. The antisymmetric symbol ϵ^{ab} is normalized so that $\epsilon^{01} = 1$.

To construct an invariant action linear in the curvature (i.e. in $\mathcal{R}$), one introduces a multiplet, $\eta_A \equiv (\eta_a, \eta_2, \eta_3)$, that transforms according to the co-adjoint representation

$$\eta_a \to (\eta_b - \eta_3 \epsilon_{bc}\theta^c)\Lambda^b{}_a, \tag{2.13}$$

$$\eta_2 \to \eta_2 - \eta_a \epsilon^a{}_b \theta^b + \frac{1}{2}\eta_3 \theta^a \theta_a, \tag{2.14}$$

$$\eta_3 \to \eta_3\,. \tag{2.15}$$

(Note that η_a may be set to zero by a gauge transformation.) The action is now simply formed by contracting η_A with $\epsilon^{\mu\nu}\mathcal{R}^A_{\mu\nu}$

$$I_g = \frac{1}{2\pi\kappa}\int d^2x \epsilon^{\mu\nu}\Big(\eta_a(\partial_\mu e^a_\nu + \omega_\mu \epsilon^a{}_b e^b_\nu) + \eta_2 \partial_\mu \omega_\nu + \eta_3(\partial_\mu a_\nu + \frac{1}{2}\epsilon_{ab}e^a_\mu e^b_\nu)\Big)\,. \tag{2.16}$$

It is easy to show [5, 7] that this *B-F* theory is equivalent to the string-inspired gravity defined by the geometrical action (2.3) once we identify $\eta_2 = 2\eta$ and $g_{\mu\nu} = e^a_\mu e_{a\nu}$. The cosmological constant, λ, is generated dynamically by the field η_3, which is fixed to be a constant by the equations of motion.
A gauge invariant description of matter requires the introduction of a new variable: the Poincaré coordinate q^a. The appearance of this additional degree of freedom is intrinsically related to the geometric structure of the Poincarè group. However, a detailed analysis of this subject goes beyond the aim of this brief review, and for a deeper analysis we refer the reader to Refs.[14, 15, 16]. Here, we only comment on some aspects useful in writing down invariant actions for matter fields.
In a Poincaré gauge theory of gravity, the connection e^a_μ cannot be really interpreted as the geometrical *zweibein*. In fact, due to the inhomogeneous nature of its transformation under the symmetry, the ensuing metric ($g_{\mu\nu}(x) = e^a_\mu \eta_{ab} e^b_\nu$) would be not gauge invariant‡. Hence, to preserve a geometrical interpretation, we need to construct a new field that plays the role of the geometrical *zweibein*.
Assuming that q^a under a gauge transformation behaves like

$$(q^U)^a = (\Lambda^{-1})^a{}_b(x)(q^b(x) + \epsilon^b{}_c \theta^c(x)), \tag{2.17}$$

the combination $E^a_\mu(q) \equiv -\epsilon^a{}_b\Big[\partial_\mu q^b(x) + \epsilon^b{}_c\Big(q^c\omega_\mu - e^c_\mu\Big)\Big]$ seems to be a good canditate. In fact it has the correct transformation law, (namely $E^a_\mu(q) = \Lambda^a_b(\alpha)E^b_\mu(q)$). Moreover, there is a gauge choice in which $E^a_\mu(q)$ can be actually identified with e^a_μ, the so-called *"physical gauge"* $q^a = 0$. When this gauge is selected, the Poincarè invariance is broken and only the Lorentz subgroup survives, so that the usual first fomalism is recovered.
In a certain sense the q^a field looks like a Higgs field in a gauge theory with symmetry breaking; its presence insures the gauge invariance, and when the unitary gauge, $q^a = 0$, is chosen the physical content of the theory is exposed.
The construction of gauge invariant actions for matter fields can be now performed in a straightforward manner. Given the geometrical Lagrangian in the first order formalism, we shall simply replace the field e^a_μ with the field $E^a_\mu(q)$ everywhere it appears. This procedure will naturally lead to the desired Lagrangian.

‡This unpleasant feature may disappear when the equations of motion impose the vanishing of the curvature fields. In such cases the symmetry may be recovered on-shell. This happens, for example, for the pure gravity described by the action (2.16).

For instance, we can consider the case of a massless scalar field, whose action in the usual first order formalism can be written as

$$\int_{\mathcal{M}} dx^2 \epsilon_{ab}\epsilon^{\mu\nu}(e^a_\mu \Pi^b \partial_\nu \phi - e^a_\mu \phi \partial_\nu \Pi^b + e^a_\mu e^b_\nu \Pi_l \Pi^l). \tag{2.18}$$

The field Π^l is an auxiliary field introduced in order to have a polynomial Lagrangian in the *zweibein* e^a_μ. The Poincarè invariant Lagrangian is now simply

$$\int_{\mathcal{M}} dx^2 \epsilon_{ab}\epsilon^{\mu\nu}(E^a_\mu(q) \Pi^b \partial_\nu \phi - E^a_\mu(q) \phi \partial_\nu \Pi^b + E^a_\mu(q) E^b_\nu(q) \Pi_l \Pi^l). \tag{2.19}$$

Finally, concerning the equations of motion derived from such Lagrangians, we have to notice that they are in general consistent only when considered together with the equations for the gravity. This feature is common to all Poincarè theories of gravity.

III The model and its classical solutions

In this section we shall investigate the string-inspired gravity coupled to a non abelian gauge field $A_\mu(x) = A^i_\mu(x) T_i$. In the geometric formulation, where the metric, the dilaton and the gauge connections are the fundamental fields, the action reads

$$\begin{aligned} I &= \frac{1}{2\pi\kappa}\int d^2x \sqrt{-g_{P}}\,(S^P(\phi)\, R(g^P) + g^{P\,\mu\nu}\,\partial_\mu\phi\partial_\nu\phi + V^P(\phi)) \\ &- \frac{1}{4}\int d^2x \sqrt{-g^P}\, W^P(\phi)\, \mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}). \end{aligned} \tag{3.1}$$

where $R(g^P)$ is the scalar curvature and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$. With respect to the usual dilaton gravity, we have added the possibility of an arbitrary dilaton potential $V^P(\phi)$ and arbitrary couplings to the dilaton ($S^P(\phi)$ and $W^P(\phi)$). If $S^P(\phi)$ is a regular invertible function for any admissible value of ϕ, the geometrical action can be connected to the gauge formulation in terms of the extended Poincarè group by means of the following field redefinition

$$g_{\mu\nu}(x) = \exp\left(\frac{1}{2}\int \frac{d\phi}{dS^P/d\phi}\right) g^P_{\mu\nu}(x) \qquad \bar{\phi} = S^P(\phi), \tag{3.2}$$

In terms of this new fields, the action takes the form

$$I = \frac{1}{2\pi\kappa}\int d^2x \sqrt{-g}(\bar{\phi} R(g) - \lambda) + \frac{1}{8}\int d^2x \sqrt{-g}\left(W(\bar{\phi})\mathrm{Tr}(\tilde{F}^2) - 2V(\bar{\phi})\right) \tag{3.3}$$

where $\tilde{F} \equiv \epsilon^{\mu\nu} F_{\mu\nu}$, and V and W are defined as

$$V(\bar{\phi}) = \frac{2\lambda}{\pi\kappa} - \frac{2}{\pi\kappa} V^P(\phi(\bar{\phi}))\exp\left(-\frac{1}{2}\int \frac{d\phi}{dS^P/d\phi}\right) \tag{3.4}$$

$$W(\bar{\phi}) = W^P(\phi(\bar{\phi}))\exp\left(\frac{1}{2}\int \frac{d\phi}{dS^P/d\phi}\right) \tag{3.5}$$

The connection with the gauge theory is now rather simple; in fact, we have

$$I_g = \frac{1}{2\pi\kappa}\int d^2x\epsilon^{\mu\nu}\Big(\eta_a(\partial_\mu e^a_\nu + \omega_\mu\epsilon^a{}_b e^b_\nu) + \eta_2\partial_\mu\omega_\nu + \eta_3(\partial_\mu a_\nu + \frac{1}{2}\epsilon_{ab}e^a_\mu e^b_\nu)\Big) \quad (3.6)$$
$$-\frac{1}{4}\int dx^2\bigg\{V(q^A\eta_A)\mathrm{det}(E(q)) - W(\eta_A q^A)\left(\mathrm{Tr}(\tilde{F}\Phi) - \frac{\mathrm{det}(E(q))}{2}\mathrm{Tr}(\Phi\Phi)\right)\bigg\}$$

where $q^A\eta_A$ is the gauge invariant combination $\eta_a q^a + \eta_2 + \frac{1}{2}\eta_3 q_a q^a$, and $\mathrm{det}(E(q)) = -1/2\epsilon_{ab}\epsilon^{\mu\nu}E^a_\mu(q)E^b_\nu(q)$. The auxiliary field $\Phi = \Phi^i T_i$ has been introduced in order to have a polynomial Lagrangian. The equivalence between the two actions (3.3) and (3.6) can be easily shown by comparing the equations of motion in the two theories. In the gauge-theoretical formulation the equations of motion read

$$\delta\eta_a \to \epsilon^{\mu\nu}(\partial_\mu e^a_\nu + \omega_\mu\epsilon^a{}_b e^b_\nu) = \frac{\pi\kappa}{2}q^a\Big[\frac{\partial V}{\partial(\eta^A q_A)}\mathrm{det}(E(q)) - \frac{\partial W}{\partial(\eta^A q_A)}\mathcal{T}\Big], \quad (3.7)$$

$$\delta\eta_2 \to \epsilon^{\mu\nu}\partial_\mu\omega_\nu = \frac{\pi\kappa}{2}\left[\frac{\partial V}{\partial(\eta^A q_A)}\mathrm{det}(E(q)) - \frac{\partial W}{\partial(\eta^A q_A)}\mathcal{T}\right], \quad (3.8)$$

$$\delta\eta_3 \to \epsilon^{\mu\nu}(\partial_\mu a_\nu + \frac{1}{2}\epsilon_{ab}e^a_\mu e^b_\nu) = \frac{\pi\kappa}{2}q^l q_l\left[\frac{\partial V}{\partial(\eta^A q_A)}\mathrm{det}(E(q)) - \frac{\partial W}{\partial(\eta^A q_A)}\mathcal{T}\right], \quad (3.9)$$

$$\delta e^a \to \partial_\mu\eta_a + \epsilon_{ab}\eta^b\omega_\mu + \epsilon_{ab}e^b_\mu\eta_3 = -\frac{\pi\kappa}{2}\left(V(\eta_A q^A) + \frac{W(\eta_A q^A)}{2}\mathrm{Tr}(\Phi\Phi)\right)\epsilon_{ab}E^b_\mu(q), \quad (3.10)$$

$$\delta\omega_\mu \to \partial_\mu\eta_2 + \epsilon_{ab}\eta^a e^b_\mu = -\frac{\pi\kappa}{2}\left(V(\eta_A q^A) + \frac{W(\eta_A q^A)}{2}\mathrm{Tr}(\Phi\Phi)\right)\epsilon_{ab}E^a_\mu(q)q^b, \quad (3.11)$$

$$\delta a_\mu \to \partial_\mu\eta_3 = 0, \quad (3.12)$$

$$\delta\Phi \to \tilde{F} - \mathrm{det}(E(q))\Phi = 0, \quad (3.13)$$

$$\delta A_\mu \to \mathcal{D}_\mu(\Phi W(\eta_A q^A)) = 0, \quad (3.14)$$

where $\mathcal{D}_\mu$ is the covariant derivative constructed with the Yang-Mills field A_μ, and $\mathcal{T}$ stands for

$$\mathcal{T} = \left(\mathrm{Tr}(\tilde{F}\Phi) - \frac{1}{2}\mathrm{det}(E(q))\mathrm{Tr}(\Phi\Phi)\right). \quad (3.15)$$

The auxiliary field Φ can be now eliminated by using (3.13). In fact we have

$$\Phi = \frac{\tilde{F}}{\mathrm{det}(E(q))}, \quad (3.16)$$

which in turn implies

$$\mathrm{Tr}(\Phi\Phi) = \frac{\mathrm{Tr}(\tilde{F}^2)}{\mathrm{det}(E^a(q))^2} \quad \text{and} \quad \mathcal{T} = \frac{1}{2}\frac{\mathrm{Tr}(\tilde{F}^2)}{\mathrm{det}(E(q))}\ . \quad (3.17)$$

Moreover, from Eq.(3.14) it is straightforward to show that the quantity

$$Q \equiv \frac{\mathrm{Tr}(\tilde{F}^2)}{\mathrm{det}(E(q))^2}W(\eta^A q_A)^2 \quad (3.18)$$

is constant (x-independent). Using (3.16)-(3.18), the equations for gravity become

$$\epsilon^{\mu\nu}(\partial_\mu e^a_\nu + \omega_\mu \epsilon^a{}_b e^b_\nu) = \frac{\pi\kappa}{2} q^a \left[\frac{\partial V}{\partial(\eta^A q_A)} + \frac{\partial W}{\partial(\eta^A q_A)} \frac{Q}{2W(\eta^A q_A)^2}\right] \det(E(q)), \tag{3.19}$$

$$\epsilon^{\mu\nu}\partial_\mu \omega_\nu = \frac{\pi\kappa}{2} \left[\frac{\partial V}{\partial(\eta^A q_A)} + \frac{\partial W}{\partial(\eta^A q_A)} \frac{Q}{2W(\eta^A q_A)^2}\right] \det(E(q)), \tag{3.20}$$

$$\epsilon^{\mu\nu}(\partial_\mu a_\nu + \frac{1}{2}\epsilon_{ab} e^a_\mu e^b_\nu) = \frac{\pi\kappa}{2} q^l q_l \left[\frac{\partial V}{\partial(\eta^A q_A)} + \frac{\partial W}{\partial(\eta^A q_A)} \frac{Q}{2W(\eta^A q_A)^2}\right] \det(E(q)), \tag{3.21}$$

$$\partial_\mu \eta_a + \epsilon_{ab}\eta^b \omega_\mu + \epsilon_{ab} e^b_\mu \eta_3 = -\frac{\pi\kappa}{2}\left(V(\eta_A q^A) + \frac{Q}{2W(\eta^A q_A)}\right)\epsilon_{ab} E^b_\mu(q), \tag{3.22}$$

$$\partial_\mu \eta_2 + \epsilon_{ab}\eta^a e^b_\mu = -\frac{\pi\kappa}{2}\left(V(\eta_A q^A) + \frac{Q}{2W(\eta_A q^A)}\right)\epsilon_{ab} E^a_\mu(q) q^b, \tag{3.23}$$

$$\partial_\mu \eta_3 = 0. \tag{3.24}$$

Let us introduce an effective dilaton potential defined by

$$\hat{V}_Q(\eta^A q_A) = V(\eta_A q^A) + \frac{Q}{2W(\eta_A q^A)}\,. \tag{3.25}$$

With this choice, the previous set of equations takes a simpler form

$$\epsilon^{\mu\nu}(\partial_\mu e^a_\nu + \omega_\mu \epsilon^a{}_b e^b_\nu) = \frac{\pi\kappa}{2} q^a \frac{\partial \hat{V}}{\partial(\eta^A q_A)} \det(E(q)), \tag{3.26}$$

$$\epsilon^{\mu\nu}\partial_\mu \omega_\nu = \frac{\pi\kappa}{2} \frac{\partial \hat{V}}{\partial(\eta^A q_A)} \det(E(q)), \tag{3.27}$$

$$\epsilon^{\mu\nu}(\partial_\mu a_\nu + \frac{1}{2}\epsilon_{ab} e^a_\mu e^b_\nu) = \frac{\pi\kappa}{2} q^l q_l \frac{\partial \hat{V}}{\partial(\eta^A q_A)} \det(E(q)), \tag{3.28}$$

$$\partial_\mu \eta_a + \epsilon_{ab}\eta^b \omega_\mu + \epsilon_{ab} e^b_\mu \eta_3 = -\frac{\pi\kappa}{2}\hat{V}_Q(\eta_A q^A)\epsilon_{ab} E^b_\mu(q), \tag{3.29}$$

$$\partial_\mu \eta_2 + \epsilon_{ab}\eta^a e^b_\mu = -\frac{\pi\kappa}{2}\hat{V}_Q(\eta_A q^A)\epsilon_{ab} E^a_\mu(q) q^b, \tag{3.30}$$

$$\partial_\mu \eta_3 = 0, \tag{3.31}$$

which are the equations of motion for gravity in absence of the gauge field. Therefore the effect of A^i_μ is only to change the shape of the dilaton potential from $V(\eta^A q_A)$ to $\hat{V}_Q(\eta^A q_A)$. Combining now Eq.(3.26) with Eq.(3.27), we can easily derive that

$$\partial_\mu E^a_\nu(q) - \partial_\nu E^a_\mu(q) + \omega_\mu \epsilon^a{}_b E^b_\nu(q) - \omega_\nu \epsilon^a{}_b E^b_\mu(q) = 0\ , \tag{3.32}$$

namely the geometrical *zweibein* $E^a_\mu(q)$ is torsionless. Notice that this property, which is fundamental if we want to have only a metric theory of gravity, does not hold for the gauge connection e^a_μ (see Eq.(3.26)). (Actually e^a_μ becomes torsionless only in the physical gauge $q^a = 0$, see Eq.(3.26) again.) This confirms what we stated in the previous section.
From now on, we focus our attention on the classical solutions of the reduced system (3.26)–(3.31). First of all we note that Eq.(3.31) requires η_3 to be a constant, and we call its value "λ" to get agreement with the geometric description in Eq.(3.3). Moreover, we can also neglect Eq.(3.28) because it simply fixes the potential a_μ, which does not play a role in the discussion of the geometry.

Combining Eqs.(3.29)-(3.31), it is straightforward to show that

$$\partial_\mu(\eta_A q^A) = -\lambda\epsilon_{ab}\left(q^a + \frac{\eta^a}{\lambda}\right)E^b_\mu. \tag{3.33}$$

and

$$\partial_\mu(\eta_A\eta^A) = -\pi\kappa\lambda\hat{V}_Q(\eta_A q^A)\epsilon_{ab}\left(q^a + \frac{\eta^a}{\lambda}\right)E^b_\mu = \pi\kappa\hat{V}_Q(\eta_A q^A)\partial_\mu(\eta_A q^A) = \partial_\mu\left(\pi\kappa J_Q(\eta_A q^A)\right)$$

$$\Longrightarrow \quad \eta_A\eta^A = \xi + \pi\kappa J_Q(\eta_A q^A) \tag{3.34}$$

where $J_Q(x)$ is a x-primitive of the function $\hat{V}_Q(x)$, ξ is a constant, and $\eta_A\eta^A$ is the gauge invariant combination $\eta_a\eta^a - 2\eta_2\eta_3$.
Taking the covariant derivative ∇_ν of Eq.(3.33), we get, with the help of Eq.(3.29),

$$\nabla_\nu\partial_\mu(\eta_A q^A) = -\lambda g_{\mu\nu}(q) + \frac{\pi\kappa}{2}\hat{V}_Q(\eta_A q^A)g_{\mu\nu}(q). \tag{3.35}$$

Note that, importantly, in this equation both the metric $g_{\mu\nu}(q)$ and the covariant derivative ∇_μ are constructed out of the geometrical *zweibein* $E^a_\mu(q)$.
Upon selecting the conformal gauge $E^a_\mu(q) = \delta^a_\mu e^{\sigma(x)}$[§], Eq.(3.35) takes the form

$$\partial_\mu\partial_\nu(\eta_A q^A) - \partial_\nu(\eta_A q^A)\partial_\mu\sigma - \partial_\mu(\eta_A q^A)\partial_\nu\sigma - \eta_{\mu\nu}\partial^\lambda(\eta_A q^A)\partial_\lambda\sigma = [\frac{\pi\kappa}{2}\hat{V}_Q(\eta_A q^A) - \lambda]e^{2\sigma}\eta_{\mu\nu}. \tag{3.36}$$

The component $++$ and $--$ ($x^+ = x + t$ and $x- = x - t$) of this equation can be, now, casted in the following way

$$\partial_+(e^{-2\sigma}\partial_+(\eta_A q^A)) = 0 \qquad \partial_-(e^{-2\sigma}\partial_-(\eta_A q^A)) = 0, \tag{3.37}$$

which are very easy to solve

$$\partial_-(\eta_A q^A) = e^{2\sigma}f(x^+)\ , \qquad \partial_+(\eta_A q^A) = e^{2\sigma}g(x^-)\ . \tag{3.38}$$

Here, $f(x^+)$ and $g(x^-)$ are two arbitrary functions, which can be set to 1 by using the residual diffeomorphism invariance present in the conformal gauge ($x^+ \to x^+(\tilde{x}^+)$ and $x^- \to x^-(\tilde{x}^-)$). With this choice the Eqs.(3.38) collapse to

$$\frac{\partial(\eta_A q^A)}{\partial t} = 0\ , \qquad \frac{\partial(\eta_A q^A)}{\partial x} = e^{2\sigma}\ . \tag{3.39}$$

Now, with the help of Eq.(3.33), Eq.(3.34) can be rewritten as follows

$$g^{\mu\nu}\partial_\mu(\eta_A q^A)\partial_\nu(\eta_A q^A) + 2\lambda(\eta_A q^A) + \pi\kappa J_Q(\eta_A q^A) + \xi = 0. \tag{3.40}$$

In conformal gauge this equation, due to Eq.(3.39), reduces to

$$\frac{\partial(\eta_A q^A)}{\partial x} + 2\lambda(\eta_A q^A) + \pi\kappa J_Q(\eta_A q^A) + \xi = 0, \tag{3.41}$$

[§]Notice that this gauge is always available, due to the diffeomorphism and Lorentz invariance

which can be integrated easily with respect to $\eta_A q^A$, giving

$$x = -\int^{\eta_A q^A} dy \frac{1}{2\lambda y + \pi\kappa J_Q(y) + \xi}. \tag{3.42}$$

This equation fixes implicitely $\eta_A q^A$ in terms of the coordinate x. In turn, from Eq.(3.39), we can compute the conformal factor $\sigma(x)$. Given these two quantities the geometry is completely determined.
It is easy to show that no further constraint arises from the remaining equations. In fact Eq.(3.27) is implied by the Eqs.(3.39) and (3.41), once the condition of vanishing torsion (3.32) is taken into account. Finally, Eqs.(3.29) and (3.30) simply determine e^a_μ and η_2, if we fix $\eta_a = 0$ by using the invariance under Poincarè translations. The Poincarè coordinate q^a is, instead, determined by solving the equation $E^a_\mu(q) \equiv -\epsilon^a{}_b \left[\partial_\mu q^b(x) + \epsilon^b{}_c \left(q^c \omega_\mu - e^c_\mu\right)\right] = e^\sigma \delta^a_\mu$.
The last step is the construction of the gauge field. Combining the equations of motion with the gauge invariance, we can always choose a solution of the form

$$F^i_{\mu\nu} = \frac{f^i}{2W(\eta_A q^A)} \det(E(q)) \epsilon_{\mu\nu} \tag{3.43}$$

where f^i is a constant vector with the property $f^i f_i = Q$.
As a closing remark on the analysis at the classical level, we notice that our results are consistent with (and generalize to the nonabelian case) those of Ref.[12], where coupling of dilaton gravity to a $U(1)$ gauge field was investigated in the framework of the geometrical formulation.

IV Quantization

We now turn to the quantization of our model. We begin by recording the Lagrange density that is the starting point of the analysis

$$\begin{aligned} \mathcal{L} = & \frac{1}{2\pi\kappa}\epsilon^{\mu\nu}\Big(\eta_a(\partial_\mu e^a_\nu + \omega_\mu \epsilon^a{}_b e^b_\nu) + \eta_2 \partial_\mu \omega_\nu + \eta_3(\partial_\mu a_\nu + \frac{1}{2}\epsilon_{ab} e^a_\mu e^b_\nu)\Big) \\ & -\frac{1}{4} V(q^A \eta_A) \det(E(q)) + \frac{1}{4} W(\eta_A q^A) \left(\mathrm{Tr}(\tilde{F}\Phi) - \frac{\det(E(q))}{2} \mathrm{Tr}(\Phi\Phi)\right) \end{aligned} \tag{4.1}$$

This can be rewritten in a way more clearly exposing the symplectic structure as follows

$$\begin{aligned} \mathcal{L} = & \frac{1}{2\pi\kappa}\left(\eta_a \dot{e}^a_1 + \eta_2 \dot{\omega}_1 + \eta_3 \dot{a}_1\right) + \frac{1}{4}\left(V(\eta_A q^A) + W(\eta_A q^A)\frac{1}{2}\mathrm{Tr}(\Phi\Phi)\right) E_{a1} \dot{q}^a \\ & + e^a_0 G_a + \omega_0 G_2 + a_0 G_3 + \frac{1}{4}\mathrm{Tr}(W(\eta_A q^A)\Phi \dot{A}_1) - \frac{1}{4}\mathrm{Tr}(A_0 \mathcal{D}_1(W(\eta_A q^A)\Phi)) \end{aligned} \tag{4.2}$$

where spatial, but not temporal, integration by parts has been carried out freely, and G_a, G_2, G_3 are the gravitational gauge generators

$$G_a \equiv \frac{1}{2\pi\kappa}\left(\eta'_a + \epsilon_a{}^b \eta_b \omega_1 + \eta_3 \epsilon_{ab} e^b_1\right) + \epsilon_a{}^b p_b \tag{4.3}$$

$$G_2 \equiv \frac{1}{2\pi\kappa}\left(\eta'_2 + \eta_a \epsilon^a{}_b e^b_1\right) - q^a \epsilon_a{}^b p_b \tag{4.4}$$

$$G_3 \equiv \frac{1}{2\pi\kappa}\eta'_3 \ . \tag{4.5}$$

The symbol p_a in Eqs.(4.3), (4.4) and (4.5) stands for the expression

$$\frac{1}{4}\left(V(\eta_A q^A) + \frac{1}{2}W(\eta_A q^A)\mathrm{Tr}(\Phi\Phi)\right) E_{a1}, \tag{4.6}$$

which is the coefficient of $\dot{q}^a$ in the Lagrangian. It is obviously convenient to promote p_a to a momentum conjugate to q_a, by introducing a Lagrange multiplier u^a that enforces the explicit form of p_a Analogously, it is convenient to promote the coefficient of $\dot{A}_1$ to a momentum Π conjugate to A_1.

$$\begin{aligned}\mathcal{L} \;=\;& \frac{1}{2\pi\kappa}\left(\eta_a \dot{e}_1^a + \eta_2 \dot{\omega}_1 + \eta_3 \dot{a}_1\right) + p_a \dot{q}^a + e_0^a G_a + \omega_0 G_2 + a_0 G_3 \\ & + \mathrm{Tr}(\Pi \dot{A}_1) + \mathrm{Tr}(A_0 \mathcal{D}_1 \Pi) + u^a \left[p_a - \frac{1}{4}\left(V(\eta_A q^A) + 8\frac{\mathrm{Tr}(\Pi^2)}{W(\eta_A q^A)}\right) E_{a1}\right]\end{aligned} \tag{4.7}$$

The symplectic structure identifies the canonical coordinates as e_1^a, ω_1, a_1, A_1, and q^a, while their respective canonical momenta are $\frac{\eta_a}{2\pi\kappa}$, $\frac{\eta_2}{2\pi\kappa}$, $\frac{\eta_3}{2\pi\kappa}$, Π, and p_a. The Hamiltonian is a superposition of the gravitational gauge constraints and Yang-Mills constraints; the Lagrange multipliers e_0^a, ω_0, a_0 enforce the vanishing of the gravitational gauge generators, u^a enforces the vanishing of

$$C_a \equiv \left[p_a - \frac{1}{4}\left(V(\eta_A q^A) + 8\frac{\mathrm{Tr}(\Pi^2)}{W(\eta_A q^A)}\right) E_{a1}\right] , \tag{4.8}$$

and A_0 enforces the vanishing (Gauss constraint) of

$$\mathcal{G} \equiv \mathcal{D}_1 \Pi \; . \tag{4.9}$$

Using the Poisson brackets implied by the symplectic structure, one verifies that the algebra of constraints closes; they are first-class. The Yang-Mills generators follow the familiar Lie algebra of the Yang-Mills group, which in components reads

$$[\mathcal{G}_a(x), \mathcal{G}_b(y)]_{PB} = f_{abc}\mathcal{G}_c(x)\; \delta(x-y), \tag{4.10}$$

and the gravitational gauge generators follow the Lie algebra

$$[G_a(x), G_b(y)]_{PB} = \epsilon_{ab} G_3(x)\; \delta(x-y) \tag{4.11}$$

$$[G_a(x), G_2(y)]_{PB} = \epsilon_{ab} G_b(x)\; \delta(x-y)\; , \tag{4.12}$$

where a common time argument has been suppressed. Quantization consists of replacing Poisson brackets by commutators. We proceed following Ref.[9], i.e. we exploit the features of the Schrödinger functional representation to postpone questions of the quantum nature of the constraint algebra (4.11)-(4.12). We seek wave functionals Ψ, in the Schrödinger representation, that are solutions of the functional differential equations corresponding to the requirement of vanishing constraints. As done in Ref.[9], we do not well-order operators in the constraints at intermediate steps of the calculation; the ordering is stipulated only at the end, when a constraint is taken to act on the wave functional. We work in "momentum" space for the metric and the Yang-Mills variables, *i.e.* Ψ depends on η_a, η_2, η_3, Π, and q^a while e_1^a, ω_1, a_1, A_1, and p_a are realized as the functional derivatives

$$\begin{gathered} e_1^a \sim 2\pi\kappa i \frac{\partial}{\partial \eta_a}\;, \quad \omega_1 \sim 2\pi\kappa i \frac{\partial}{\partial \eta_2}\;, \quad a_1 2 \sim 2\pi\kappa i \frac{\partial}{\partial \eta_3}\;, \\ A_1 \sim \frac{1}{i}\frac{\delta}{\delta \Pi}\;, \quad p_a \sim \frac{1}{i}\frac{\delta}{\delta q^a}\;, \end{gathered} \tag{4.13}$$

Having clarified the objectives and methodology of our investigation, we can proceed in the investigation of the quantum mechanical theory. We begin by observing that the $G_3 = 0$ constraint simply requires that the wave functional depends only on the constant part of η_3, which we call λ (in analogy with the classical theory, where it corresponds to the cosmological constant). We also notice that

$$\frac{\eta^a}{\lambda} G_a - G_2 = \frac{M'}{4\pi\kappa\lambda} + \left(q^a + \frac{\eta^a}{\lambda}\right) \epsilon_a{}^b p_b \tag{4.14}$$

where M is the gauge-invariant combination¶

$$M = \eta_a \eta^a - 2\eta_2\eta_3 \ . \tag{4.15}$$

Eq.(4.14) implies that, as a result of the $G_a = 0$ and $G_2 = 0$ constraints, in the space of physical wave functionals the following operatorial relation holds

$$\frac{M'}{4\pi\kappa\lambda} = -\left(q^a + \frac{\eta^a}{\lambda}\right) \epsilon_a{}^b p_b \ . \tag{4.16}$$

In light of the above observations, it is convenient to shift some of the (functional) variables used to describe the wave functional. The variables q^a can be conveniently replaced by the shifted variables

$$\rho^a \equiv q^a + \eta^a/\lambda \ , \tag{4.17}$$

which respond only to Lorentz gauge transformations (they are translation and $U(1)$ invariant). p_a can then be taken as conjugate to ρ^a. Moreover, η_2 can be shifted by $\eta_a\eta^a/(2\lambda)$, so that $-2\lambda\eta_2$ is replaced by the gauge invariant variable M. Correspondingly $\frac{1}{2\pi\kappa}\omega_1$, the coordinate conjugate to η_2, is renamed $2\lambda\Pi_M$, with Π_M conjugate to M.
With these redefinitions, one finds that wave functionals satisfying the gravitational gauge constraints take the form (also using the notations $\hat{\rho}^a \equiv \rho^a/\rho$ and $\rho \equiv \sqrt{\rho^a\rho_a}$)

$$\Psi = \delta(\eta_3 - \lambda)\, e^{i\Omega}\, e^{i\widetilde{\Omega}}\, \widetilde{\Psi}(M, \eta_3, \Pi, \rho^2) \tag{4.18}$$

where Ω is the Kirillov-Kostant 1-form on the coadjoint orbit of the extended Poincaré group

$$\Omega = \frac{1}{4\pi\kappa\,\lambda} \int \epsilon^{ab}\eta_a d\eta_b \ , \tag{4.19}$$

and

$$\widetilde{\Omega} = \frac{1}{4\pi\kappa\lambda} \int d\hat{\rho}^a \epsilon_{ab} \hat{\rho}^b M \ . \tag{4.20}$$

Once Eq.(4.14) is taken into account, the fact that (4.18) solves the gravitational gauge constraints can be easily checked in complete analogy with corresponding analyses presented in Refs.[5, 7, 9]. The η_a-independence of $\widetilde{\Psi}$ can be traced back to the $G_a = 0$ constraint, whereas the vanishing of G_2 causes $\widetilde{\Psi}$ to depend on ρ_a only through its magnitude ρ.
The structure of $\widetilde{\Psi}$ is further constrained by the $C_a = 0$ and $\mathcal{G} = 0$ requirements, which we have not yet imposed. In particular, the $\mathcal{G} = 0$ constraint implies $(\Pi^2)' = 0$, *i.e.* the wave

¶In the classical theory in absence of the Yang-Mills fields, M is constant and corresponds to the "black hole" mass[5, 7, 9].

functional depends only on the constant part of Π^2, which we call $Q^2/16$ to be in agreement with the conventions of sec. 3.
Instead of imposing directly the constraint C_a, we equivalently consider the combination

$$H_a \equiv C_a + \frac{1}{4\lambda}\hat{V}_Q(m)\epsilon^a{}_b G^b = p^a + \frac{1}{4}\hat{V}_Q(m)[\epsilon^a{}_b \rho'^b + \frac{2\pi\kappa}{\lambda}(p^a + 2\lambda^2\rho^a\Pi_M)] \,, \tag{4.21}$$

where $\hat{V}_Q$ is the effective dilaton potential defined in the previous section, and we also introduced the notation

$$m \equiv \eta_A q^A = \frac{\lambda^2\rho^2 - M}{2\lambda} \ . \tag{4.22}$$

To proceed it is convenient to decompose this constraint in its radial and angular part, namely

$$H_a = \epsilon_{ab}\frac{\rho^b}{\rho^2}(\epsilon^{lm}\rho_l H_m) + \frac{\rho_a}{\rho^2}(\rho^l H_l) \tag{4.23}$$

with

$$(\rho^l H_l) = \rho_a p^a + \frac{1}{4}\hat{V}_Q(m)[\epsilon_{ab}\rho^a{\rho'}^b + \frac{2\pi\kappa}{\lambda}(\rho_a p^a + 2\lambda^2\rho^2\Pi_M)]. \tag{4.24}$$

$$(\epsilon^{lm}\rho_l H_m) = -\frac{1}{4\pi\kappa\lambda}[M' - \pi\kappa\hat{V}_Q(m)m'] \tag{4.25}$$

In Eq.(4.25) we have used Eq.(4.16) to eliminate the combination $\epsilon_{ab}\rho^a p^b$.
Taking into account that $\rho_a p^a \widetilde{\Omega} = 0$, the radial part (4.24) leads to the following constraint on $\widetilde{\Psi}$

$$\rho_a p^a + \frac{\pi\kappa}{2\lambda}\hat{V}_Q(m)(\rho_a p^a + 2\lambda^2\rho^2\Pi_M). \tag{4.26}$$

The structure of Eq.(4.26) suggests that it might be convenient to shift the M dependence in the remaining $\widetilde{\Psi}$ functional by the term $-M(1+1/2\lambda) + \lambda\rho^2/2$, which is equivalent to a canonical transformation from M, Π_M, and p_a to $m, \Pi_m = -2\lambda\Pi_M$, and $\Pi_a = p_a + 2\lambda^2\rho_a\Pi_M$ (ρ^a is unaffected). In terms of the variables m and ρ^2, the constraint (4.26) on $\widetilde{\Psi}$ takes the form

$$\left[1 - \frac{\pi\kappa}{2\lambda}\hat{V}_Q(m)\right]\frac{\delta\widetilde{\Psi}}{\delta\rho^2} + \lambda\frac{\delta\widetilde{\Psi}}{\delta m} = 0. \tag{4.27}$$

It is then easy to see that Eq.(4.27) implies that the wave functional only depends on the following combination of the m and ρ^2 variables:

$$\xi \equiv \lambda^2\,\rho^2 - 2\,\lambda\,m - \pi\kappa J_Q(m) = (M - \pi\kappa J_Q(m)) \ , \tag{4.28}$$

where $J_Q(x)$, as in the sec. 3, is a x-primitive of the function $\hat{V}_Q(x)$.
Subsequently from (4.25) it follows that only the constant part of $M - \pi\kappa J_Q(m)$ can appear nontrivially in the wave functional.
Finally, we observe that the solutions of the $\mathcal{G}=0$ constraint associated to the gauge field A are of the form

$$e^{-i\Omega_\Pi} f_{GI}(\Pi) \quad \text{with} \quad \mathit{\Omega}_{\mathit{\Pi}} = \int < K, g_{K\mathit{\Pi}}\, dg^{-1}_{K\mathit{\Pi}} > \ , \tag{4.29}$$

where we use the index "GI" to indicate that f_{GI} is a gauge-invariant functional of Π (*i.e.* depends only on the characters of the Yang-Mills group), $<,>$ is the invariant inner product

of the Lie algebra of the Yang-Mills group, K is any fixed element of the Lie algebra, and $g_{K\Pi}$ is a group element such that $\Pi = g_{K\Pi}^{-1} K g_{K\Pi}$. For example, if the Yang-Mills group is $SU(2)$ (generators σ_1, σ_2, σ_3), the reader can easily check that $\mathcal{G}$ vanishes on the functional

$$e^{-i\Omega_\Pi^{SU(2)}} f_{GI}(\Pi) \quad \text{with} \quad \Omega_\Pi^{SU(2)} = \int \Pi_2 \frac{\Pi_1 \, d\Pi_3 - \Pi_3 \, d\Pi_1}{\Pi_1^2 + \Pi_3^2} \ , \tag{4.30}$$

which corresponds to $K = \sigma_3$ and, accordingly,

$$g_{K\Pi} = e^{i\theta_1 \sigma_1} e^{i\theta_2 \sigma_2} \quad \text{with} \quad \theta_1 = -\frac{1}{2} \arcsin(\Pi_2) \ , \quad \theta_2 = \frac{1}{2} \arcsin\left(\frac{\Pi_1}{\sqrt{\Pi_1^2 + \Pi_3^2}} \right) \ . \tag{4.31}$$

The above analysis of the $C_a = 0$ and $\mathcal{G} = 0$ constraints leads to the following final result for $\widetilde{\Psi}$

$$\widetilde{\Psi} = \delta(\xi') \, \delta((\Pi^2)') \, e^{-i\Omega_\Pi} \, \widetilde{\widetilde{\Psi}}_{GI}(\xi, \Pi) \ , \tag{4.32}$$

which together with Eq.(4.18) gives the sought physical wave funtional.

It is useful to consider some limiting cases of our result.

If $V = W = 0$, the starting Lagrangian reduces to the one of free gravity. It is not difficult to check that the correct wave functional is recovered. In fact, $J_Q(m) \to 0$ in this limit, and only the dependence on the constant part of M survives in agreement with reference [7, 9].

If $W = 0$ but $V \neq 0$, we reproduce, by using the gauge formulation, the results about the most general dilaton gravity obtained in Ref.[10]. Moreover it is interesting to notice that for $V(x) = x^2$ the model is equivalent to the R^2 gravity with the constraint of vanishing torsion.

If $V = 0$ but $W \neq 0$, one can realize, by varying W, a family of couplings involving metrics that can be obtained by conformal rescaling of the Cangemi-Jackiw gauge metric. In particular, the case $W =$ constant corresponds to Yang-Mills fields minimally coupled to the gauge metric, whereas $W(x) = \exp[\int (d\phi/2)(dS^P/d\phi)^{-1}]$ corresponds to Yang-Mills fields minimally coupled to the physical metric[8].

V Closing Remarks

Our analysis of string-inspired gravity coupled to Yang-Mills fields prompts several considerations.

Let us start by observing that the gauge theoretical Cangemi-Jackiw formulation of string-inspired gravity has led indeed to a very natural description of the coupling to Yang-Mills fields. We reproduced and generalized several results known in the geometrical formulation, by showing that, in the momentum representation, the constraints could be straightforwardly enforced with the help of a series of Kirillov-Kostant phases.

It should also be noticed that, whereas in the realm of the geometrical formulation the class of theories here considered appears to be completely general, in the gauge-theoretical formulation one can assume more complicated structures for the potentials V and W, in which they depend on the additional two field variables ξ and η_3. [We remind the reader that ξ and η_3 are numbers in the geometrical formulation, whereas they are independent scalar fields in the gauge-theoretical formulation.] Roughly speaking, a nontrivial dependence on ξ allows for both dynamical torsion and curvature, while a nontrivial dependence on η_3 "turns on" the fields a_μ associated to the central extension. It would be interesting to investigate these more general scenarios.

Our results also indicate that Yang-Mills fields in two dimensions have no dynamical degrees of freedom even when coupled to dilaton gravity. As shown by Eq.(4.28), they only affect the geometry by modifying the relation between the variable M and the constant mode ξ characterizing the wave functional ($\xi = M$ in pure gravity). The solvability of the diffeomorphism constraints can be traced back to this topological nature of the Yang-Mills fields, and the fact that, in such a context, these constraints can be genuinely traded for the gauge-theoretical constraints of the extended Poincaré group. The difficulties encountered in Ref.[9], where the coupling of a scalar field to dilaton gravity was investigated, can be interpreted as a consequence of the disruption of the topological structure caused by the dynamical degree of freedom of the scalar field.

We also notice that in the context here considered, unlike the case of dilaton gravity coupled to point particles[8] no spurious divergences resulted from the use of the Poincaré variables. This supports the interpretation[8] of the divergences encountered in the point-particle case as a purely technical difficulty, originating from the fact that the description of N point particles requires the introduction of N sets of Poincaré variables with singularities associated to the configurations with overlapping particle positions. The description of Yang-Mills fields coupled to dilaton gravity requires the introduction of only one set of Poincaré (field) variables.

We close by reemphasizing that our analysis should be considered only as a first step toward the challenging objective of a fully consistent quantization of the gravity Yang-Mills system. By following the approach of Refs.[5, 7, 9, 10], we have postponed the issue of the quantum nature of the constraint algebra, and the problem of defining a consistent funtional measure in the space of *physical* functionals that we identified. This level of analysis has allowed us to make a preliminary investigation of the structure of the gravity Yang-Mills system, leading in particular to the observation of several differences between this system and the previously investigated cases of pure gravity[5], gravity coupled to point particles[7, 8], and gravity coupled to scalar matter fields[9]. We hope that our results will be useful and will provide motivation for future studies in which the quantum nature of the problem be fully explored.

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